Chapter 4.
A Taxonomy of Classical Randomized Experiments

presented by Insung Kong

Seoul National University

2022 07/07

To be classified as classical randomized experiments, assignment mechanism should be

1. individualistic.
2. probabilistic.
3. unconfounded.
4. has a known functional form.

In this chapter we introduce **four specific examples** of classical randomized assignment mechanisms.

Notations

- $\mathbb{W}^+$: subset of the set of possible values for $\boldsymbol{W}$ with positive probability.
- $e(x)$: propensity score which is strictly between 0 and 1.

Definition 4.1 (Bernoulli Trial) A Bernoulli trial is a classical randomized experiment with an assignment mechanism such that the assignments for all units are independent.

Theorem 4.1 (Assignment Mechanism for a Bernoulli Trial) If the assignment mechanism is a Bernoulli trial, then

$$\Pr(\boldsymbol{W} \mid \boldsymbol{X}, \boldsymbol{Y}(0), \boldsymbol{Y}(1)) = \prod_{i=1}^{N} \left[e\left(X_i\right)^{W_i} \cdot \left(1 - e\left(X_i\right)\right)^{1-W_i} \right],$$

where $e(x)$ is the propensity score, which must be strictly between 0 and 1 for all i, implying $\mathbb{W}^{+} = \{0,1\}^{N}$.

Definition 4.2 (Completely Randomized Experiment)
A completely randomized experiment is a classical randomized experiment with an assignment mechanism satisfying

$$\mathbb{W}^{+} = \left\{ \boldsymbol{W} \in \mathbb{W} \mid \sum_{i=1}^{N} W_i = N_{\mathrm{t}} \right\}$$

for some preset $N_{\mathrm{t}} \in \{1, 2, \ldots, N-1\}$.

- All $\binom{N}{N_{\mathrm{t}}}$ assignment vectors in $\mathbb{W}^{+}$ are equally likely.
- $\Pr(\boldsymbol{W} \mid \boldsymbol{X}, \boldsymbol{Y}(0), \boldsymbol{Y}(1)) = \begin{cases} \binom{N}{N_{\mathrm{t}}}^{-1} & \text{if } \sum_{i=1}^{N} W_i = N_{\mathrm{t}}, \\ 0 & \text{otherwise} \end{cases}$

- $B_i \in \{1, \ldots, J\}$ indicate the block or stratum of the i^{th} unit.
- $N(j)$ $(N_t(j))$: the number of (treated) units in i^{th} block.

Definition 4.3 (Stratified Randomized Experiment)

A stratified randomized experiment with J blocks is a classical randomized experiment with an assignment mechanism satisfying

$$\mathbb{W}^+ = \left\{ \boldsymbol{W} \in \mathbb{W} \mid \sum_{i:B_i=j}^{N} W_i = N_t(j), \text{ for } j = 1, 2, \ldots, J \right\},$$

and

$$\Pr(\boldsymbol{W} \mid \boldsymbol{X}, \boldsymbol{Y}(0), \boldsymbol{Y}(1)) = \begin{cases} \prod_{j=1}^{J} \begin{pmatrix} N(j) \\ N_t(j) \end{pmatrix}^{-1} & \text{if } \boldsymbol{W} \in \mathbb{W}^+ \\ 0 & \text{otherwise} \end{cases}$$

for some preset $N_t(j)$ such that $N_j > N_t(j) > 0$, for $j = 1, \ldots, J$.

Definition 4.4 (Paired Randomized Experiment)
A paired randomized experiment is a stratified randomized experiment with $N(j) = 2$ and $N_t(j) = 1$ for $j = 1, \ldots, N/2$, so that

$$\mathbb{W}^{+} = \left\{ \boldsymbol{W} \in \mathbb{W} \mid \sum_{i:B_i=j}^{N} W_i = 1, \ \text{for } j = 1, 2, \ldots, N/2 \right\}$$

and

$$\Pr(\boldsymbol{W} \mid \boldsymbol{X}, \boldsymbol{Y}(0), \boldsymbol{Y}(1)) = \begin{cases} 2^{-N/2} & \text{if } \boldsymbol{W} \in \mathbb{W}^{+}, \\ 0 & \text{otherwise.} \end{cases}$$